MA/MSCMT-01

December - Examination 2016

M.A./M.Sc. (Previous) Mathematics Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A $8 \times 2 = 16$

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define derived subgroup.

(ii) Define Euclidean ring.

(iii) Define dual base.

(iv) Define normal extension of a field.

(v) Define eigen vector.

(vi) Write Pythogoras theorem in inner product space.

(vii) Write Bessel's inequality.

(viii) Define orthogonal linear transformation.

Section - B $\quad 4 \times 8 = 32$

Note: Section 'B' contain 08 Short Answer type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) State and prove class equation of a group.

3) Let R be a Euclidean ring with a Euclidean valuation d. Then prove that $d(1)$ is minimal among all $d(a)$ for non zero $a \in$ R and $u \in$ R is a unit if and only if $d(u) = d(1)$.

4) If F C K C E are fields with [E : K] and [K : F] finite, then prove that E/F is finite extension and

$$[E : F] = [E : K]\,[K : F]$$

5) Prove that an $n \times n$ matrix A over a field F is invertible if and only if rank (A) = n.

6) Let A = [aij] be any $n \times n$ matrix over a field F, then prove that

$$\det(A) = \sum_{\sigma \in S_n} \in(\sigma)\, a_{\sigma(1)1}\, a_{\sigma(2)2} \,\ldots\ldots\ldots\ldots\, a_{\sigma(n)n}$$

7) Prove that every orthonormal set of vector is a linearly independent set in an inner product space.

8) Let t : V $\rightarrow$ V^1 be a linear transformation and V is finite dimensional, then prove that

$$\dim V = \text{rank}(t) + \text{nullity}(t).$$

9) If F is a field, then prove that every polynomial $f(x) \in F[x]$ has splitting field.

Section - C $\quad 2 \times 16 = 32$

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) (i) Prove that a group G is solvable if and only if $G(n) = \{e\}$ for some $n \in N$.

(ii) If K is a field and if $\sigma_1, \sigma_2, \dots\dots\dots\sigma_n$ are distinct automorphisms of K, then prove that it is impossible to find elements $a_1, a_2, \dots\dots\dots a_n$ not all zero in K such that $a_1\, \sigma_1(u) + a_2\, \sigma_2(u) + \dots\dots\dots + a_n\, \sigma_n(u) = 0$ for all $u \in K$.

11) Let R be a Euclidean ring. Then prove that any finitely generated R module N is the direct sum of a finite number of cyclic sub modules.

12) (i) Let $t : V \rightarrow V$ be a linear transformation from a finite dimensional vector space V to itself. Assume that v_i; $i = 1, 2, \dots\dots n$ are distinct eigenvectors of t corresponding to distinct eigenvalues λ_i; $i = 1, 2, \dots\dots n$. Then prove that $\{v_1, v_2, \dots\dots v_n\}$ is a linearly independent set.

(ii) If V be a finite dimensional inner product space and W be its any subspace. Then prove that V is the direct sum of W and $W^{\perp}$.

13) State and prove principle axis theorem.